Math 55 Discussion problems 28 Feb

1. Suppose that for every pair of cities in a country there is a direct one-way road connecting them in one direction or the other. Use mathematical induction to show that there is a city that can be reached from every other city either directly or via exactly one other city.
2. Prove that if $A_1, A_2, ..., A_n$ and B are sets, then $(A_1 - B) \cap (A_2 - B) \cap \cdots \cap (A_n - B) = (A_1 \cap A_2 \cap \cdots \cap A_n) - B$.
3. Let f_n be the nth Fibonacci number. Prove that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$ when n is a positive integer.
4. Give a recursive definition of the functions max and min so that $\max(a_1, a_2, ..., a_n)$ and $\min(a_1, a_2, ..., a_n)$ are the maximum and minimum of the n numbers $a_1, a_2, ..., a_n$, respectively.
5. (a) Give a recursive definition of the function ones(s), which counts the number of ones in a bit string s.

   (b) Use structural induction to prove that ones(st) = ones(s) + ones(t).